Module 2-1 tutorial
Solutions

Austroads

The stochastic nature of traffic behaviour

Question 1. Assume a bus stop that multiple bus lines share. The number of buses arriving at the bus stop is on average 1 bus per minute. Dwell time of each bus is stochastic and takes an average of 20 seconds. Find:

a) The probability that the bus stop is empty.

b) The probability of having 1 bus at the bus stop.

c) The average number of buses waiting for the bus stop.

d) The average total time of buses to leave the bus stop.

Solution to Question 1.

The arrival rate is 1 bus per minute; r=1. The departure rate is 3 buses per minute; s=3. So ρ=r/s=0.33.

a) P_0=1-ρ=0.67. This means on average 67% of the time the bus stop is empty.

b) P_1=ρ(1-ρ)=0.22. This means on average 22% of the time there is (exactly) one bus at the stop.

c) E(m)= ρ^2/(1-ρ)=0.17. This means the average number of waiting buses excluding the one at the bus stop is 0.17.

d) $E(\tau) = \frac{1}{s-r} = \frac{1}{3-1} = 0.5\ minute = 30\ s$. *The 30 seconds on average include 20 seconds of dwell time and 10 second of waiting for the bus in front to leave the bus stop.*

Question 2. A motorway ramp holds 10 vehicles. A ramp metering system controls the entrance of vehicles into the motorway such that on average 1 vehicle enters every 6 seconds. On average 1 vehicle arrives at the ramp every 8 seconds. Determine

a) The percent of the time ramp is empty.

b) The percent of the time the ramp spills back.

c) The expected queue size on the ramp.

Solution to Question 2.

$r = \frac{3600}{8} = 450\ \frac{veh}{h}$; $s = \frac{3600}{6} = 600\ \frac{veh}{h}$; $\rho = \frac{r}{s} = 0.75$

a) $P_0 = 1 - \rho = 1 - 0.75 = 0.25 = 25\%$

b) $Pr(n > 10) = \rho^{11} = 0.75^{11} = 0.042 = 4.2\%$

c) $E(n) = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3\ vehicle$