Module 2-2 tutorial

Solutions

Austroads

Fundamental speed-flow-density relationships

Question 1. Four vehicles are observed travelling along a long straight section of a motorway. Two of the vehicles are travelling at 60 km/h, one is travelling at 50 km/h and the slowest is travelling at 30 km/h. Clearly, a variable of interest would be the average speed for this stream of traffic. Consider a loop detector records the speed of the vehicles as they pass.

a) Calculate an average of those spot speeds. Is this the time or space mean speed?

b) Now imagine a one-kilometre section of the motorway and the time each vehicle takes to pass that section has been recorded. For each of the vehicle speeds given above, determine how long it will take them to pass the one-kilometre section.

c) Average those times and use that value to get an average speed. Is this the time or space mean speed?

<u>Solution to Question 1</u>.

a) The average of those spot speeds is (60+60+50+30)/4=50 km/h. This is time mean speed.

b) The times to pass the one-kilometre section for each of the vehicles are:

Vehicles travelling at 60 km/r will take 60 s: (1 km / 60 km/h) = 1/60 h = 60 s

Vehicle travelling at 50 km/h will take 72 s.

Vehicle travelling at 30 km/h will take 120 s.

c) Average travel time = (60×2+72+120)/4 = 78 s.

Average speed = distance/average travel time

Distance = 1km

Hence, average speed = 1 km / 78 s = 46.15 km/h. And this is space mean speed.

To confirm; 4 / ((1/60)+ (1/60)+ (1/50)+ (1/30)) = 46.15 km/h.

For any traffic stream, space mean speed is always less than or equal to time mean speed. So, using time mean speed underestimates average travel time. This is a typical error in travel time estimation.

Question 2. You are using some loop detector data in a project, but you suspect the data are not accurate because the occupancy is always 10%. So, you plan to investigate the data by taking some speed samples with a radar gun at the location of interest. During your 5-min observation, there are 40 vehicles with speed 40 km/h, 50 vehicles at 30 km/h and 10 vehicles at 15 km/h. The average vehicle length is about 6 m and the length of the loop detector is 2 m.

Calculate the density based on the loop detector data and the observation data and compare. Do you think the loop detector data are reliable?

Timestamp	Vehicle count	Occupancy
08:00:00	100	10%
08:05:00	140	10%
08:10:00	90	10%
08:15:00	120	10%
08:20:00	70	10%
08:25:00	130	10%

Solution to Question 2.

From observations:

$$q = \frac{(40 + 50 + 10)\ veh}{5\ min} = 1200\ veh/h$$

$$v_t = \frac{\sum(n_i v_i)}{N} = \frac{40 \times 40 + 50 \times 30 + 10 \times 15}{40 + 50 + 10} = 32.5\ km/h$$

$$v_s = \frac{N}{\sum(n_i \frac{1}{v_i})} = \frac{40 + 50 + 10}{40 \times \frac{1}{40} + 50 \times \frac{1}{30} + 10 \times \frac{1}{15}} = 30\ km/h$$

$$k = \frac{q}{v_s} = \frac{1200}{30} = 40\ veh/km$$

From Loop detector data:

$k = \frac{o}{(L_v + L_d)} = \frac{0.1}{(6+2)/1000} = 12.5\ veh/km$. *The loop detector data is flawed and underestimate the density.*

Question 3. The fundamental diagram of a motorway is as follows: $v = v_{\mathrm{f}}\left(1 - \frac{k}{k_{\mathrm{j}}}\right)$; where v is the space mean speed km/h, k is density veh/km, v_{f} is the free-flow speed km/h and k_{j} is the jam density veh/km. A section of the motorway is known to have free-flow speed as 110 km/h and jam density as 120 veh/km. Find the capacity, q_{max}, of the section.

Solution to Question 3.

a) First, we must find the q-k fundamental diagram.

$q = kv$ *and* $v = v_f\left(1 - \frac{k}{k_j}\right)$*. We have:* $q = kv_f\left(1 - \frac{k}{k_j}\right)$

The capacity q_{max} *is the maximum flow occurs when density is* k_{cap}*. That is, we are interested in the value of* k *at which* $\frac{dq}{dk} = 0$*.Therefore* $\frac{dq}{dk} = 0 = v_f\left(1 - \frac{2k}{k_j}\right)$*.*

Thus, either $v_f = 0$ *or* $\left(1 - \frac{2k}{k_j}\right) = 0$*. Because the free-flow speed* (v_f) *cannot be zero, it implies* $\left(1 - \frac{2k}{k_j}\right) = 0$*. Hence* $k_{cap} = \frac{k_j}{2}$*. We have:* $q = kv_f\left(1 - \frac{k}{k_j}\right)$ *and* $k_{cap} = \frac{k_j}{2}$ *so,* $q_{max} = \frac{k_j}{2}v_f\left(1 - \frac{1}{2}\right) = \frac{v_f k_j}{4}$*.*

$v_f = 110$ *and* $k_j = 120$ *so* $q_{max} = 3300$ *veh/h.*